NORTH SYDNEY GIRLS HIGH SCHOOL

2007
TRIAL HSC EXAMINATION

Mathematics Extension 1

General Instructions
- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top. Submit one bundle. The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number:____________________ **Teacher:**________________

Student Name:____________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

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Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 Marks) Use a SEPARATE writing booklet — **Marks**

(a) Indicate the region on the number plane satisfied by $y \geq |2x-5|$. **2**

(b) Differentiate $5x\tan^{-1}x$ with respect to x. **2**

(c) Solve the inequality $\dfrac{x+4}{x-3} \leq 2$. **3**

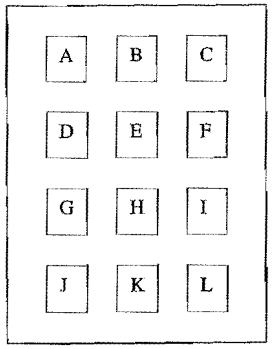
(d) Use the substitution $u = 16 - x^2$ to find $\int_0^2 x\sqrt{16-x^2}\,dx$. **3**

(e) The interval AB has endpoints $A(3,2)$ and $B(4,5)$. Find the coordinates of the point P which divides the interval AB externally in the ratio of $3:4$. **2**

Question 2 (12 Marks) Use a SEPARATE writing booklet — **Marks**

(a) Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\sin 4x}{5x} + \cos x\right)$. **2**

(b) The security lock of a building has 12 buttons labelled as shown.



Each person using the lock is given a 4 letter access code.

(i) How many different access codes are possible if the letters can be repeated and their order is important? **1**

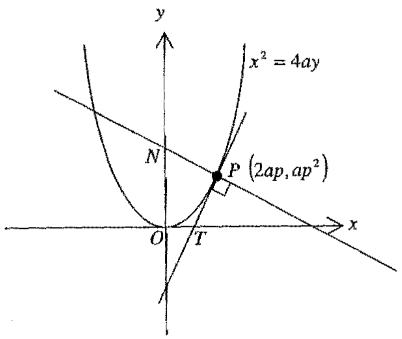
(ii) How many different access codes are possible if letters cannot be repeated and their order is important? **1**

(iii) Now suppose that the lock operates by holding 4 buttons down together, so that the order is not important. How many different access codes are possible? **1**

Question 2 continues on page 5

Question 2 (continued) **Marks**

(c)



The diagram shows the graph of the parabola $x^2 = 4ay$. The tangent to the parabola at $P(2ap,\ ap^2)$ cuts the x-axis at T. The normal to the parabola at P cuts the y-axis at N.

(i) Show that the equation of the tangent at P is $y = px - ap^2$ and find the coordinates of T. **2**

(ii) Show that the coordinates of N are $(0,\ a(p^2+2))$. **2**

(iii) Let M be the midpoint of NT. Find the Cartesian equation of the locus of M and describe this locus geometrically. **3**

Question 3 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) When the polynomial $P(x)$ is divided by $x^2 - 5x + 6$, the remainder is $5x + 2$. What is the remainder when $P(x)$ is divided by $x - 2$? **3**

(b) Show that the expression **3**

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}; \quad \sin\theta \neq 0,\ \cos\theta \neq 0$$

is independent of θ.

(c) Find the derivative of $\log_e(\cos x)$. Hence find the area enclosed by the curve $y = \tan x$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{3}$. **3**

(d) A plane is observed in the air at point P, 500 metres above ground level. An observer standing at A observes the plane at an angle of elevation of 32°. A second observer at B observes the angle of elevation of the plane to be 24°. A is due East of Q. B is $S42°E$ of Q. **3**

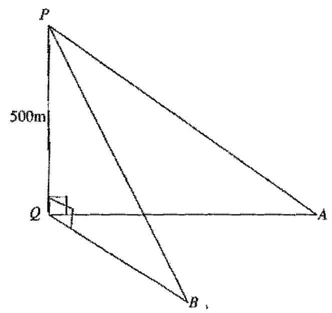
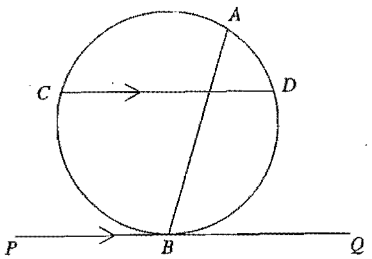


Diagram not to scale

Calculate the distance from A to B. Answer to the nearest metre.

Question 4 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) AB and CD are intersecting chords of a circle. CD is parallel to the tangent to the circle at B.



(i) Copy this diagram into your writing booklet.

(ii) Prove that AB bisects $\angle CAD$ **4**

(b) Using the substitutions for $t = \tan\frac{\theta}{2}$, solve the equation **4**

$3\cos\theta + 5\sin\theta = 4,\ 0 \le \theta \le 2\pi$

(c) Use the principle of mathematical induction to prove that **4**

$4^n + 14$ is a multiple of 6 for $n \ge 1$.

Question 5 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Solve the polynomial equation $x^3 - 12x^2 + 12x + 80 = 0$ if the roots of this equation are in arithmetic progression. **3**

(b) The function $g(x)$ is given by $g(x) = \sin^{-1}x + \cos^{-1}x,\ 0 \le x \le 1$

(i) Find $g'(x)$ **1**

(ii) Sketch the graph of $y = g(x)$. **2**

(c) An egg at room temperature, 20°C is placed in a saucepan of boiling water which is maintained at 100°C. When the egg has been in the boiling water for t minutes the internal temperature of the egg is T°C. The rate at which the internal temperature of the egg rises is proportional to the difference between the egg's internal temperature and that of the boiling water i.e. T satisfies the equation:

$$\frac{dT}{dt} = k(T - 100), \text{ where } k \text{ is a constant.}$$

(i) Show that $T = 100 + Ae^{kt}$ satisfies the equation. **1**

(ii) The internal temperature of the egg rises to 60°C after 10 minutes. Find the values of A and k. **2**

(iii) How long does it take for the internal temperature of the egg to reach 90°C? **1**

(iv) What would happen to the internal temperature of the egg if the egg was left in the boiling water indefinitely? Justify your answer. **2**

Question 6 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) Grain is poured at a constant rate of 6 cubic metres per minute. It forms a conical pile, with the semi-vertical angle of the cone equal to 60°. The height of the pile is h metres and the radius of the base is r metres.

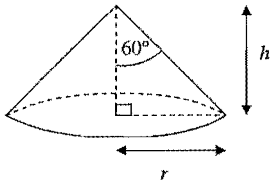


Diagram not to scale

(i) Show that $r = \sqrt{3}h$. **1**

(ii) Find an expression for the volume of the pile. **1**

(iii) Hence find the rate at which the height of the pile is increasing when the height of the pile is 3 metres. **2**

(b) Consider the function $f(x) = (x-1)^2 - 4,\ x \geq 0$

(i) Sketch the function showing clearly any intercepts and the coordinates of its vertex, using the same scale on the x and y axes. **1**

(ii) What is the largest domain, containing $x = 2$, for which the function has an inverse function $f^{-1}(x)$? **1**

(iii) What is the domain of the inverse function $f^{-1}(x)$? **1**

(iv) Sketch the graph of $y = f^{-1}(x)$ on the same set of axes as part (i). **1**

(v) Find the equation of the inverse function as a function of x. **2**

(vi) Find the x-coordinate of the point of intersection of the two curves $y = f(x)$ and $y = f^{-1}(x)$. **2**

Question 7 (12 Marks) Use a SEPARATE writing booklet **Marks**

(a) The region enclosed by the curve $y = \sin^{-1} x$ and the y-axis between $x = 0$ and $x = \frac{\sqrt{3}}{2}$ is rotated about the y-axis to form a solid. Find the volume of this solid. **5**

(b) Let each different arrangement of all the letters of the word "DELETED" be considered a word.

(i) How many words are possible altogether? **1**

(ii) In how many ways can the three Es be together? **1**

(iii) Show that there are 240 ways that two Es can be together and one separate. **3**

(iv) What is the probability that all the Es are apart? **2**

End of paper

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

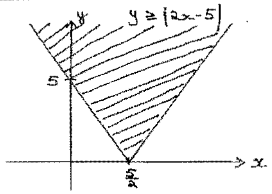
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1

a)



b) $\frac{d}{dx}\left(5x\tan^{-1}x\right)$

$= 5x \times \frac{1}{1+x^2} + \tan^{-1}x \times 5$

$= \frac{5x}{1+x^2} + 5\tan^{-1}x$

c) $\frac{x+4}{x-3} \leq 2 \qquad (\times(x-3)^2)$ $(x \neq 3)$

$(x+4)(x-3) \leq 2(x-3)^2$

$x^2+x-12 \leq 2(x^2-6x+9)$

$x^2+x-12 \leq 2x^2-12x+18$

$0 \leq x^2-13x+30$

$x^2-13x+30 \geq 0$

$(x-3)(x-10) \geq 0$

$\begin{cases} x < 3 \\ x \geq 10 \end{cases}$

d) $u = 16-x^2$

$\frac{du}{dx} = -2x$

$x = -\frac{1}{2}\frac{du}{dx}$

$x = 2, \quad u = 12$

$x = 0, \quad u = 16$

$\int_0^2 x\sqrt{16-x^2}\,dx$

$= \int_{16}^{12} u^{\frac{1}{2}} \times -\frac{1}{2}\frac{du}{dx}\,dx$

$= -\frac{1}{2}\int_{16}^{12} u^{\frac{1}{2}}\,du$

$= -\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{16}^{12}$

$= \frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_{12}^{16}$

$= \frac{1}{3}\left[u^{\frac{3}{2}}\right]_{12}^{16}$

$= \frac{1}{3}\left(16^{\frac{3}{2}} - 12^{\frac{3}{2}}\right)$

$= \frac{1}{3}\left(64 - (2\sqrt{3})^3\right)$

$= \frac{1}{3}\left(64 - 24\sqrt{3}\right)$

$= \frac{64-24\sqrt{3}}{3}$

$= \frac{64}{3} - 8\sqrt{3}$

e) $m : n = -3 : 4$

$x = \frac{nx_1+mx_2}{n+m} \qquad y = \frac{ny_1+my_2}{n+m}$

$x = \frac{(4)(3)+(-3)(4)}{(4)+(-3)} \qquad y = \frac{(4)(2)+(-3)(5)}{1}$

$x = \frac{12-12}{1} \qquad y = \frac{8-15}{1}$

$x = 0 \qquad y = -7$

$P\ (0, -7)$

Question 2

a) $\lim_{x \to 0} \frac{\sin 4x}{4x} \times \frac{4}{5} + \cos x$

$= 1 \times \frac{4}{5} + 1$

$= 1\frac{4}{5}$

b) (i) 12^4 (20736)

(ii) ${}^{12}P_4$ (11880)

(iii) ${}^{12}C_4$ (495)

c) i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$

$x = 2ap$, $\frac{dy}{dx} = \frac{2ap}{2a}$

$= p$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

For $T \Rightarrow y = 0$

$0 = px - ap^2$

$px = ap^2$

$x = ap$

$\therefore T(ap, 0)$

(ii) Gradient of normal is $-\frac{1}{p}$

Eqn of normal at P

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

For $N \Rightarrow x = 0$

$y - ap^2 = -\frac{1}{p}(-2ap)$

$y - ap^2 = 2a$

$y = ap^2 + 2a$

$= a(p^2+2)$

$\therefore N(0, a(p^2+2))$

(iii) M

$x = \frac{ap+0}{2}$ $y = \frac{0 + a(p^2+2)}{2}$

$x = \frac{ap}{2}$ $y = \frac{a(p^2+2)}{2}$

$\Rightarrow p = \frac{2x}{a}$

$y = \frac{a}{2}\left(\left(\frac{2x}{a}\right)^2 + 2\right)$

$= \frac{a}{2}\left(\frac{4x^2}{a^2} + 2\right)$

$y = \frac{2x^2}{a} + a$

$ay = 2x^2 + a^2$

$2x^2 = ay - a^2$

$2x^2 = a(y - a)$

$x^2 = \frac{1}{2}a(y - a)$

$4A = \frac{1}{2}a$

$A = \frac{1}{8}a$

This is a parabola with Vertex $(0, a)$ and focal length $\frac{a}{8}$ units.

Question 3

a) $P(x) = (x^2 - 5x + 6)Q(x) + (5x+2)$

$= (x-3)(x-2)Q(x) + (5x+2)$

$P(2) = (2-3)(2-2)Q(2) + (5(2)+2)$

$= 0 + 12$

$= 12$

b) $\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta}$

$= \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$

$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

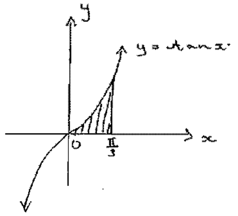
$= 2$

which is independent of θ

c) $\frac{d}{dx}(\log_e(\cos x))\,dx$

$= \frac{-\sin x}{\cos x}$

$= -\tan x$



c) $A = \int_0^{\frac{\pi}{3}} \tan x \, dx$

$= \left[-\log_e(\cos x)\right]_0^{\frac{\pi}{3}}$

$= \left(-\log_e \cos\frac{\pi}{3}\right) - \left(-\log_e \cos 0\right)$

$= \left(-\log_e \frac{1}{2}\right) - \left(-\log_e 1\right)$

$= -\log_e \frac{1}{2} + 0$

$= \log_e\left(\frac{1}{2}\right)^{-1}$

$= \log_e 2$

d) $\frac{500}{QA} = \tan 32^\circ$

$QA = \frac{500}{\tan 32^\circ}$

$\frac{500}{QB} = \tan 24^\circ$

$QB = \frac{500}{\tan 24^\circ}$

$AB^2 = QA^2 + QB^2 - 2(QA)(QB)\cos 48^\circ$

$AB \doteqdot 835.986\ldots$

Distance is approximately 836m

Question 4

a) Join AC, AD, DB

let $\angle DBQ = \theta$

$\angle BAD = \theta$ angle between a tangent & a chord at the point of contact is equal to the angle in the alternate segment.

$\angle CDB = \theta$ alternate angles equal $CD \parallel PQ$

$\angle CAB = \angle CDB$ angles standing on the same arc are equal.

$\therefore \angle CAB = \angle BAD$ both θ

$\therefore$ AB bisects $\angle CAD$.

b) $3\cos\theta + 5\sin\theta = 4$

$3 \times \frac{1-t^2}{1+t^2} + 5 \times \frac{2t}{1+t^2} = 4$

$3(1-t^2) + 5(2t) = 4(1+t^2)$

$3 - 3t^2 + 10t = 4 + 4t^2$

$7t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{100-28}}{14}$

$= \frac{10 \pm \sqrt{72}}{14}$

$\tan\frac{\theta}{2} = \frac{5 \pm 3\sqrt{2}}{7}$

$\frac{\theta}{2} = 0.922\ldots,\ 0.108\ldots$

$\theta = 1.85^c,\ 0.22^c$

c) Show true for $n = 1$

$4^1 + 14 = 18$

which is divisible by 6.

Assume true for $n = k$ ie

$4^k + 14 = 6M$ where M is an integer

Prove true for $n = k+1$ ie

$4^{k+1} + 14 = 6N$ where N is an integer

$LHS = 4^{k+1} + 14$

$= 4.4^k + 14$

$= 4(4^k + 14) - 3 \times 14$

$= 4(6M) - 42$

$= 6(4M - 7)$

which is a multiple of 6

Since true for $n=1$ and proved true for $n=k+1$ when assumed true for $n=k$, it follows to be true for $n = 1+1 = 2$, $n = 2+1 = 3$ etc all positive integral values of $n \geq 1$.

Question 5

a) let the roots be

$\alpha - d,\ \alpha,\ \alpha + d$

$\sum\alpha = -\frac{b}{a}$

$3\alpha = 12$

$\alpha = 4$ *

$\therefore 4-d,\ 4,\ 4+d$

$\sum\alpha\beta\gamma = -\frac{d}{a}$

$4(4-d)(4+d) = -80$

$16 - d^2 = -20$

$d^2 = 36$

$d = \pm 6$

(± 6 will produce same roots)

$\therefore$ roots are $-2, 4, 10$

b) (i) $g(x) = \sin^{-1}x + \cos^{-1}x$

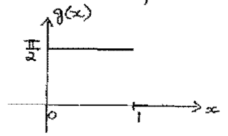
$g'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}$

$= 0$

(ii) $g(x)$ is a constant

$g(0) = \sin^{-1}0 + \cos^{-1}0$

$= \frac{\pi}{2}$, $0 \leq x \leq 1$



c) (i) $T = 100 + Ae^{kt}$

$\frac{dT}{dt} = k(Ae^{kt})$

$Ae^{kt} = T - 100$

$\frac{dT}{dt} = k(T - 100)$

(ii) $t = 0$, $T = 20°C$

$20 = 100 + Ae^0$

$A = -80$

$t = 10$, $T = 60°C$

$60 = 100 - 80e^{k \times 10}$

$-40 = -80e^{10k}$

$e^{10k} = \frac{1}{2}$

$10k = \log_e \frac{1}{2}$

$k = \frac{1}{10}\log_e\frac{1}{2}$

$(\doteqdot -0.069\ldots)$

(iii) $T = 90$, $t = ?$

$90 = 100 - 80e^{kt}$

$-10 = -80e^{kt}$

$\frac{1}{8} = e^{kt}$

$kt = \log_e\frac{1}{8}$

$t = \frac{1}{k}\log_e\frac{1}{8}$

$= \frac{10}{\log_e\frac{1}{2}}\log_e\frac{1}{8}$

$= 30$

it takes about 30mins

(iv) $T = 100 - 80e^{kt}$

$k < 0$

$\therefore t \to \infty$, $e^{kt} \to 0$

$-80e^{kt} \to 0$ from below

$100 - 80e^{kt} \to 100$ from below

egg temperature $\to 100°C$

Question 6

a) $\frac{r}{h} = \tan 60^\circ$

$r = h \tan 60^\circ$

$r = h\sqrt{3}$

$r = \sqrt{3}h$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(\sqrt{3}h)^2 h$

$= \pi h^3$

$\frac{dV}{dh} = 3\pi h^2$

$\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$

$6 = 3\pi h^2 \frac{dh}{dt}$

$h=3$, $6 = 3\pi(3)^2\frac{dh}{dt}$

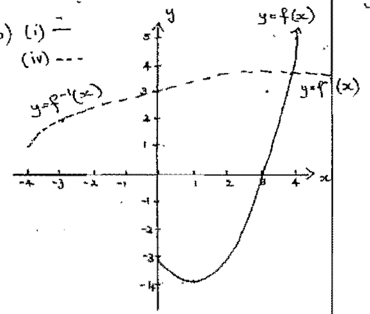
$\frac{dh}{dt} = \frac{6}{27\pi}$

$= \frac{2}{9\pi}$

rate of increase is $\frac{2}{9\pi}$ m/min.

b) (i) —

(iv) - - -



$f(x) = (x-1)^2 - 4$, $x \geq 0$

Vertex $(1,-4)$

(ii) $x \geq 1$ domain $f(x)$

(iii) domain $f^{-1}(x)$

$x \geq -4$

(range $f(x)$: $y \geq -4$)

(iv) see sketch

(v) $y = (x-1)^2 - 4$

inverse function

$x = (y-1)^2 - 4$

$(y-1)^2 = x+4$

$y - 1 = \pm\sqrt{x+4}$

$y = 1 \pm \sqrt{x+4}$, $y \geq 1$

$y = 1 + \sqrt{x+4}$

(vi) $f(x)$ and $f^{-1}(x)$ will intersect on $y = x$

$(x-1)^2 - 4 = x$

$x^2 - 2x + 1 - 4 = x$

$x^2 - 3x - 3 = 0$

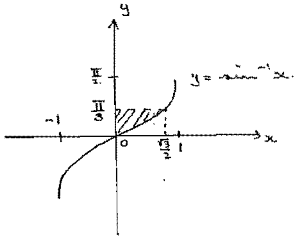
$x = \frac{3 \pm \sqrt{9+12}}{2}$

$= \frac{3 \pm \sqrt{21}}{2}$, $x \geq 1$

$x = \frac{3+\sqrt{21}}{2}$

Question 7

a)



$y = \sin^{-1} x$

$\Rightarrow x = \sin y$

$V = \pi\int_0^{\frac{\pi}{3}} (\sin y)^2\, dy$

$= \pi\int_0^{\frac{\pi}{3}} \sin^2 y\, dy$

($\cos 2y = 1 - 2\sin^2 y$

$2\sin^2 y = 1 - \cos 2y$

$\sin^2 y = \frac{1}{2}(1 - \cos 2y)$)

$V = \pi \times \frac{1}{2}\int_0^{\frac{\pi}{3}} 1 - \cos 2y\, dy$

$= \frac{\pi}{2}\left[y - \frac{1}{2}\sin 2y\right]_0^{\frac{\pi}{3}}$

$= \frac{\pi}{2}\left(\left(\frac{\pi}{3} - \frac{1}{2}\sin\frac{2\pi}{3}\right) - \left(0 - \frac{1}{2}\sin 0\right)\right)$

$= \frac{\pi}{2}\left(\left(\frac{\pi}{3} - \frac{1}{2}\times\frac{\sqrt{3}}{2}\right) - (0)\right)$

$= \frac{\pi}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

$= \frac{\pi^2}{6} - \frac{\sqrt{3}\pi}{8}$

b)

(i) $\frac{7!}{2!3!} = 420$

(ii) (EEE) D L T D

$\frac{5!}{2!} = 60$

(iii)

E E _ (E) (E) (E) (E)	4
_ E E _ (E) (E) (E)	3
(E) _ E E _ (E) (E)	3
(E) (E) _ E E _ (E)	3
(E) (E) (E) _ E E _	3
(E) (E) (E) (E) _ E E	4

2 Es together 1 Separate.

$2 \times 4 \times \frac{4!}{2} + 4 \times 3 \times \frac{4!}{2}$

$= 240$

(iv) complement of (ii) + (iii)

$= 420 - (60 + 240)$

$= 120$

$P(\text{all Es apart}) = \frac{120}{420}$

$= \frac{12}{42}$

$= \frac{2}{7}$

(b) (i) No restriction: 7 letters with 2 Ds and 3 E's ie $\frac{7!}{2!3!}=420$

(ii) All Es together ie (EEE) X X X X where X represents a letter from D L T D.

There are 5 objects with 2 repetitions ie $\frac{5!}{2!}=60$

(iii) Let X represent a letter from D L T D and S represents a space.

ALTERNATIVE 1

Arrange the letters D L T D – this can be done in $\frac{4!}{2!}=12$ ways.

Now S D S L S T S D S is one such possible arrangement.
To keep EE and E separate, we have to choose 2 spaces to insert them.
This can be done in ${}^5C_2=10$ ways.
Then the EE and E could be swapped in 2 ways.

Total $\frac{4!}{2!}\times {}^5C_2\times 2=12\times 10\times 2=240$

ALTERNATIVE 2

Case 1: (EE X E) X X X ie group the EE and E with one letter between them. There are 4 choices for the X between EE and E and then we have 4 objects with 2 repetitions.

ie $4\times\frac{4!}{2!}=48$. However the EE and E could be swapped making it $4\times\frac{4!}{2!}\times 2=96$

Case 2: (EE X X E) X X ie group the EE and E with two letters between them. There are ${}^4P_2=12$ arrangements of the X between EE and E and then we have 3 objects with 2 repetitions.

ie $12\times\frac{3!}{2!}=36$.

However the EE and E could be swapped making it $12\times\frac{3!}{2!}\times 2=72$

Case 3: (EE X X X E) X ie group the EE and E with three letters between them. There are ${}^4P_3=24$ arrangements of the X between EE and E and then we have 2 objects with 2 repetitions

ie $24\times\frac{2!}{2!}=24$.

However the EE and E could be swapped making it $24\times\frac{2!}{2!}\times 2=48$

Case 4: (EE X X X X E) ie group the EE and E with four letters between them. There are $\frac{{}^4P_4}{2!}=12$ arrangements of the X between EE and E, given 2 Ds. However the EE and E could be swapped making it $\frac{24!}{2!}\times 2=24$

Total $96+72+48+24=240$

ALTERNATIVE 3

Case 1: E E X (E) (E) (E) (E)
ie there are 4 possibilities for the position of the second E and then the other letters need arrangement.

ie $4\times\frac{4!}{2!}=48$

Case 2: X EE X (E) (E) (E)
ie there are 3 possibilities for the position of the second E and then the other letters need arrangement.

ie $3\times\frac{4!}{2!}=36$

Case 3: (E) X EE X (E) (E)
ie there are 3 possibilities for the position of the second E and then the other letters need arrangement.

ie $3\times\frac{4!}{2!}=36$

Case 4: (E) (E) X EE X (E)
ie there are 3 possibilities for the position of the second E and then the other letters need arrangement.

ie $3\times\frac{4!}{2!}=36$

Case 5: (E) (E) (E) X EE X
ie there are 3 possibilities for the position of the second E and then the other letters need arrangement.

ie $3\times\frac{4!}{2!}=36$

Case 6: (E) (E) (E) (E) X E E
ie there are 4 possibilities for the position of the second E and then the other letters need arrangement.

ie $4\times\frac{4!}{2!}=48$

Total $2\times 48+4\times 36=240$

(iv) **ALTERNATIVE 1**

From (ii) and (iii) the number of ways of having all the Es apart is
$420-(60+240)=120$.

So the probability of having all the Es apart is $\frac{120}{420}=\frac{2}{7}$.

ALTERNATIVE 2

Arrange the letters D L T D – this can be done in $\frac{4!}{2!}=12$ ways.

Now S D S L S T S D S is one such possible arrangement.
To keep ALL the Es separate, we have to choose 3 spaces to insert them.
This can be done in ${}^5C_3=10$ ways.

Total $\frac{4!}{2!}\times {}^5C_3=12\times 10=120$.

So the probability of having all the Es apart is $\frac{120}{420}=\frac{2}{7}$.